Post-Tensioned Concrete Analysis & Design

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Learning Objectives

- Identify how to determine long term prestress force loss and its effect on service level and ultimate strength design.
- Recognize the effects of secondary post-tensioning forces on frame construction.
- Determine member size and prestress force using the feasible region method and understand load balancing and equivalent loading.
Post-Tensioned Concrete Analysis and Design

• Common Materials & Properties / Basics
  • Common Prestressing Materials and Strengths
  • Bonded and Unbonded Systems / Construction Sequence

• General Post-Tensioned Concrete Framing Parameters
  • Span-to-Depth Ratios / Deflection
  • Range of Prestress Force and the Effective Flange
  • Classes of Prestressed Concrete Members

• Design Concept
  • Feasible Region
  • Load Balancing
  • Equivalent Load
Post-Tensioned Concrete Analysis and Design

• Steps in Post-Tensioned Concrete Design

• Example 1 – Simple Beam Design
  (Graphical Solution / Feasible Region)
  • Conventional Reinforced Concrete Solution
  • Post-Tensioned Solution

• Post-Tensioning Losses

• Example 2 – Continuous Beam Design
  (Load Balancing and Equivalent Loading)
# Post-Tensioned Concrete Analysis and Design

## Common Materials & Properties

### ASTM Standard Prestressing Strands, Wires, and Bars

<table>
<thead>
<tr>
<th>Type*</th>
<th>Nominal diameter, in.</th>
<th>Nominal area, in.²</th>
<th>Nominal weight, lb/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seven-wire strand (Grade 250)</td>
<td>1/4 (0.250)</td>
<td>0.036</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>5/16 (0.313)</td>
<td>0.058</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>3/8 (0.375)</td>
<td>0.080</td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td>7/16 (0.438)</td>
<td>0.108</td>
<td>0.367</td>
</tr>
<tr>
<td></td>
<td>1/2 (0.500)</td>
<td>0.144</td>
<td>0.490</td>
</tr>
<tr>
<td></td>
<td>(0.600)</td>
<td>0.216</td>
<td>0.737</td>
</tr>
<tr>
<td>Seven-wire strand (Grade 270)</td>
<td>3/8 (0.375)</td>
<td>0.085</td>
<td>0.290</td>
</tr>
<tr>
<td></td>
<td>7/16 (0.438)</td>
<td>0.115</td>
<td>0.390</td>
</tr>
<tr>
<td>1/2 (0.500)</td>
<td>0.153</td>
<td>0.520</td>
<td></td>
</tr>
<tr>
<td>(0.520)</td>
<td>0.167</td>
<td>0.570</td>
<td></td>
</tr>
<tr>
<td>(0.563)</td>
<td>0.192</td>
<td>0.650</td>
<td></td>
</tr>
<tr>
<td>(0.600)</td>
<td>0.217</td>
<td>0.740</td>
<td></td>
</tr>
<tr>
<td>(0.620)</td>
<td>0.231</td>
<td>0.780</td>
<td></td>
</tr>
<tr>
<td>(0.700)</td>
<td>0.294</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Prestressing wire</td>
<td>0.192</td>
<td>0.029</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>0.196</td>
<td>0.030</td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td>0.250</td>
<td>0.049</td>
<td>0.170</td>
</tr>
<tr>
<td></td>
<td>0.276</td>
<td>0.060</td>
<td>0.204</td>
</tr>
</tbody>
</table>

### Prestressing bars (Type I, plain)

<table>
<thead>
<tr>
<th></th>
<th>3/4</th>
<th>7/8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1/8</td>
<td>0.78</td>
<td>2.67</td>
<td>3.38</td>
</tr>
<tr>
<td>1-1/4</td>
<td>1.23</td>
<td>4.17</td>
<td>5.05</td>
</tr>
<tr>
<td>1-3/8</td>
<td>1.48</td>
<td>5.05</td>
<td>5.56</td>
</tr>
</tbody>
</table>

### Prestressing bars (Type II, deformed)

<table>
<thead>
<tr>
<th></th>
<th>3/4</th>
<th>7/8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.85</td>
<td>3.01</td>
<td>3.10</td>
</tr>
<tr>
<td>1-1/4</td>
<td>1.25</td>
<td>4.39</td>
<td>4.39</td>
</tr>
<tr>
<td>1-3/8</td>
<td>1.58</td>
<td>5.56</td>
<td>5.56</td>
</tr>
</tbody>
</table>

*Availability of some strand, wire, and bar sizes should be investigated in advance.

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* “Building Code Requirements for Structural Concrete (ACI 318-14)”, ACI Committee 318, 2014.
Unbonded Systems

- Unstressed during concrete placement,
- Encapsulated, sheathing tight to steel,
- Free to move within tendon sheathing,
- Ultimate strain not linearly related to section strain.
Post-Tensioned Concrete Analysis and Design

Common Materials & Properties

Bonded System

- Unstressed during concrete placement,
- Encapsulated, within oversized duct,
- Grouted after stressing,
- Maintains strain compatibility for Ultimate Strength
## Post-Tensioned Concrete Analysis and Design

### Framing Parameters

Span-to-Depth Ratio: Reinforced Concrete (Table 7.3.1.1, Table 9.3.1.1, Table 8.3.1.1)

<table>
<thead>
<tr>
<th>Support Condition</th>
<th>Slabs</th>
<th>Beams</th>
<th>2-Way Slab, No Drop Panels</th>
<th>2-Way Slab, w/ Drop Panels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Exterior Panel</td>
<td>Exterior Panel</td>
</tr>
<tr>
<td>Simple Span</td>
<td>l/20</td>
<td>l/16</td>
<td>Interior</td>
<td>Interior</td>
</tr>
<tr>
<td>One End Continuous</td>
<td>l/24</td>
<td>l/18.5</td>
<td>No Beam</td>
<td>No Beam</td>
</tr>
<tr>
<td>Both Ends Continuous</td>
<td>l/28</td>
<td>l/21</td>
<td>Beam</td>
<td>Beam</td>
</tr>
<tr>
<td>Cantilever</td>
<td>l/10</td>
<td>l/8</td>
<td>l/30</td>
<td>l/33</td>
</tr>
</tbody>
</table>

Span-to-Depth Ratio: Post-Tensioned Concrete

<table>
<thead>
<tr>
<th>Continuous Spans</th>
<th>Simple Spans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Roof</td>
</tr>
<tr>
<td>One-Way Slabs</td>
<td>l/50</td>
</tr>
<tr>
<td>2-Way Sold Slabs</td>
<td>l/45-l/48</td>
</tr>
<tr>
<td>2-Way Waffle Slabs</td>
<td>l/40</td>
</tr>
<tr>
<td>Beams</td>
<td>l/35</td>
</tr>
<tr>
<td>One-Way Joists</td>
<td>l/42</td>
</tr>
</tbody>
</table>

Approximate for members whose live load is less than the dead load.
Post-Tensioned Concrete Analysis and Design

Framing Parameters

Average Precompression

- Code Minimum – 125 psi (2 way slabs)
- Typical Lower Range
  - 150 psi – Floor Slabs
  - 200 psi – Roof Slabs
- Common Ranges
  - 250-300 psi
  - 250-400 psi beams based on RC Effective Flange
- Threshold for Significant Creep/Shrinkage Effects
  - > 500 psi (ACI 423)
  - > 500 psi not uncommon for girders and long span members.
Effective T-Beam Flange

- ACI 318-14, 6.3.2.3: “For prestressed T-beams, it shall be permitted to use the geometry provided by 6.3.2.1 and 6.3.2.2”
  - “The determination of an effective flange width for prestressed T-Beams is left to the experience and judgment of the licensed design professional.”
Drop Panels for Two-Way Plates

• ACI 318-14, 8.2.4: “A drop panel in a nonprestressed slab, where used to reduce the minimum required thickness in accordance with 8.3.1.1 or the quantity of deformed negative moment reinforcement at a support in accordance with 8.5.2.2, shall satisfy (a) and (b):…”

• No mention of requirements for prestressed slabs.
• Previous ACI editions expressly omitted this requirement for prestressed two-way slabs.
Post-Tensioned Concrete Analysis and Design

Classes of Prestress Members

<table>
<thead>
<tr>
<th>Prestressed</th>
<th>Class U</th>
<th>Class T</th>
<th>Class C</th>
<th>Nonprestressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumed behavior</td>
<td>Uncracked</td>
<td>Transition between uncracked and cracked</td>
<td>Cracked</td>
<td>Cracked</td>
</tr>
<tr>
<td>Section properties for stress calculation at service loads</td>
<td>Gross section 24.5.2.2</td>
<td>Gross section 24.5.2.2</td>
<td>Cracked section 24.5.2.3</td>
<td>No requirement</td>
</tr>
<tr>
<td>Allowable stress at transfer</td>
<td>24.5.3</td>
<td>24.5.3</td>
<td>24.5.3</td>
<td>No requirement</td>
</tr>
<tr>
<td>Allowable compressive stress based on uncracked section properties</td>
<td>24.5.4</td>
<td>24.5.4</td>
<td>No requirement</td>
<td>No requirement</td>
</tr>
<tr>
<td>Tensile stress at service loads 24.5.2.1</td>
<td>$\leq 7.5\sqrt{f'}$</td>
<td>$7.5\sqrt{f'} &lt; f, \leq 12\sqrt{f'}$</td>
<td>No requirement</td>
<td>No requirement</td>
</tr>
<tr>
<td>Deflection calculation basis</td>
<td>24.2.3.8, 24.2.4.2 Gross section</td>
<td>24.2.3.9, 24.2.4.2 Cracked section, bilinear</td>
<td>24.2.3.9, 24.2.4.2 Cracked section, bilinear</td>
<td>24.2.3, 24.2.4.1 Effective moment of inertia</td>
</tr>
<tr>
<td>Crack control</td>
<td>No requirement arrange</td>
<td>No requirement</td>
<td>24.3</td>
<td>24.3</td>
</tr>
<tr>
<td>Computation of $\Delta f_{ps}$ or $f_s$, for crack control</td>
<td>—</td>
<td>—</td>
<td>Cracked section analysis</td>
<td>M/(As x lever arm), or 2/3fy</td>
</tr>
<tr>
<td>Side skin reinforcement</td>
<td>No requirement</td>
<td>No requirement</td>
<td>9.7.2.3</td>
<td>9.7.2.3</td>
</tr>
</tbody>
</table>

*“Building Code Requirements for Structural Concrete (ACI 318-14)”, Table R24.5.2.1, ACI Committee 318, 2014.*
allowable stresses – class u

<table>
<thead>
<tr>
<th>Allowable stresses – Class U</th>
<th>psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Stresses immediately after prestress transfer (before prestress losses) shall not exceed the following:</td>
<td></td>
</tr>
<tr>
<td>a) Extreme fiber stress in compression</td>
<td>$0.60f'_{ci}$</td>
</tr>
<tr>
<td>b) Extreme fiber stress in tension except as permitted in c</td>
<td>$-3\sqrt{f'_{ci}}$</td>
</tr>
<tr>
<td>c) Extreme fiber stress in tension at ends of simply supported members. Where computed tensile stresses exceed these values, bonded auxiliary reinforcement (non-prestressed or prestressed) shall be provided in the tensile zone to resist the total tensile force in the concrete computed with the assumption of an uncracked section.</td>
<td>$-6\sqrt{f'_{ci}}$</td>
</tr>
<tr>
<td>2. Stresses at service loads (after allowance for all prestress losses) shall not exceed the following:</td>
<td></td>
</tr>
<tr>
<td>a) Extreme fiber stress in compression</td>
<td>$0.45f'_{c}$</td>
</tr>
<tr>
<td>a) Extreme fiber stress in tension in precompressed tensile zone</td>
<td>$-6\sqrt{f'_{c}}$</td>
</tr>
</tbody>
</table>
Post-Tensioned Concrete Analysis and Design

Design Concept

Active Reinforcing to Resist Tension in Concrete Section

- High Strength Prestressing Steel, $F_{pu} = 270 \, \text{ksi}$
  - Reduced Reinforcing Compared to Conventional Reinforced Concrete

- Pre-compress Section at Early Concrete Age
  - Reduces Shrinkage Cracking

- Counteract Tensile Forces due to DL and LL Moments
  - Precompression (P/A)
  - Prestressing Moment (Pe)
  - Secondary Moments in Continuous Construction
Post-Tensioned Concrete Analysis and Design

Design Concept

Live Load

Dead Load

(P within Central Kern)

\[
P/A + \frac{Pey}{I} + \frac{Md(y)}{I} = \text{Initial Stress}
\]

\[
\text{Initial Stress} + \frac{MI(y)}{I} = \text{Final Stress}
\]

Compression Stress

Limiting Final Tensile Stress
Post-Tensioned Concrete Analysis and Design

Feasible Region

Feasible Region – Limiting Stresses

• Creation of stress inequality equations based on member section properties and allowable initial and service level tensile and compressive stress limits dictated by the building code.

• 4 inequality conditions can be used to solve for:
  • $F_e$ (Final Effective Force)
  • $E_0$ (eccentricity of tendon along span)
  • $S_t$ and $S_b$ (section moduli), somewhat iteratively.

Example 1 will illustrate this method.
Post-Tensioned Concrete Analysis and Design

Load Balancing

• Selection of post-tensioning force and tendon profile such that the internal force induced by the post-tensioning offsets some percentage of the member initial load (typically dead load.)

• Tendon profile is selected to match the initial load bending diagram and the appropriate equivalent load equation is solved to obtain the tendon drape and force.

\[ M_i = f(xw, l'x) \quad \text{and} \quad M_{eq} = f(aw_{eq} l') \]

\[ \sigma_i = \frac{M_i y}{I} \quad \text{is equated with} \quad \sigma_{eq} = \frac{M_{eq} y}{I} \]

where \( x, y, a, \) and \( b \) are coefficients.

Example 2 will illustrate this method.
### Equivalent Loading

<table>
<thead>
<tr>
<th>Tendon Pattern</th>
<th>Equivalent Moment or Load</th>
<th>Equivalent Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Tendon Pattern" /></td>
<td>$M = Pe$</td>
<td><img src="image2" alt="Equivalent Loading" /></td>
</tr>
<tr>
<td><img src="image3" alt="Tendon Pattern" /></td>
<td>$M = Pe$</td>
<td><img src="image4" alt="Equivalent Loading" /></td>
</tr>
<tr>
<td><img src="image5" alt="Tendon Pattern" /></td>
<td>$M = Pe$</td>
<td><img src="image6" alt="Equivalent Loading" /></td>
</tr>
<tr>
<td><img src="image7" alt="Tendon Pattern" /></td>
<td>$N = \frac{4Pe'}{l}$</td>
<td><img src="image8" alt="Equivalent Loading" /></td>
</tr>
<tr>
<td><img src="image9" alt="Tendon Pattern" /></td>
<td>$N = \frac{Pe'}{bl}$</td>
<td><img src="image10" alt="Equivalent Loading" /></td>
</tr>
</tbody>
</table>
## Post-Tensioned Concrete Analysis and Design

### Equivalent Loading

<table>
<thead>
<tr>
<th>Tendon Pattern</th>
<th>Equivalent Moment or Load</th>
<th>Equivalent Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>( w = \frac{8Pe'}{l^2} )</td>
<td><img src="image2.png" alt="Diagram 2" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 2" /></td>
<td>( w = \frac{8Pe'}{l^2} )</td>
<td><img src="image4.png" alt="Diagram 3" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram 3" /></td>
<td>( w = \frac{4Pe'}{(0.5 - b)l^2} )</td>
<td>( w_1 = \frac{w}{b}(0.5 - b) )</td>
</tr>
<tr>
<td><img src="image6.png" alt="Diagram 4" /></td>
<td>( w = \frac{4Pe'}{(0.5 - b)l^2} )</td>
<td>( w_1 = \frac{w}{b}(0.5 - b) )</td>
</tr>
</tbody>
</table>
Post-Tensioned Concrete Analysis and Design
Design – Steps I to III

I. Determine Dimensions and Forces
   ▪ Member Loading and Span – Approximate Depth Required
   ▪ Concrete Strength (f’c) and Tendon Strength (fu)
   ▪ Effective Tendon Force (Fpe) and Limiting Eccentricities (e_o)

II. Determine Prestress Loss
    ▪ Anchorage Seating (Post-Tensioned Only)
    ▪ Elastic Shortening
    ▪ Creep and Shrinkage
    ▪ Tendon Relaxation
    ▪ Friction (Post-Tensioned Only)

III. Initial Stress Analysis
    ▪ Selfweight Dead Load Only
    ▪ Initial Forces from Prestressing Strands Before Losses, fpi
Post-Tensioned Concrete Analysis and Design

Design – Steps IV to VI

IV. Final Stress Analysis
- Selfweight Dead Load
- Superimposed Dead Load
- Live Load
- Final Effective Tendon Force, fpe

V. Ultimate Strength Analysis
- Factored Moments and Secondary Moments (Continuous Construction)
- Tendon Stress/Force at Nominal Strength, fps
- Ignore Tendon Internal Forces / Equivalent Loading
- Shear and/or Torsion

VI. Additional Checks
- Limiting Reinforcing Ratio
- Minimum Bonded Steel (unbonded tendons or two-way slabs only)
- Overstrength Against Cracking Moment (bonded or precast only)
- Deflection
  - Prismatic if Uncracked
### Post-Tensioned Concrete Analysis and Design

#### Design – Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ac</td>
<td>area of concrete cross-section (in²)</td>
</tr>
<tr>
<td>Aps</td>
<td>area of prestressing reinforcement (tendons) (in²)</td>
</tr>
<tr>
<td>As</td>
<td>area of mild reinforcing (in²)</td>
</tr>
<tr>
<td>dp</td>
<td>distance from extreme compressive fiber to centroid of prestressing reinf. (in)</td>
</tr>
<tr>
<td>d</td>
<td>distance from extreme compressive fiber to centroid of mild reinforcing (in)</td>
</tr>
<tr>
<td>e₀</td>
<td>eccentricity of the prestressing reinforcement from centroid of the cross section (in)</td>
</tr>
<tr>
<td>Fi</td>
<td>initial prestressing force before losses (kips)</td>
</tr>
<tr>
<td>Fe</td>
<td>final prestressing force after losses (kips)</td>
</tr>
<tr>
<td>fpi</td>
<td>initial stress in prestressing reinforcing before losses (ksi)</td>
</tr>
<tr>
<td>fpe</td>
<td>final stress in prestressing reinforcing after losses (ksi)</td>
</tr>
<tr>
<td>fpu</td>
<td>specified ultimate stress in prestressing reinforcing (ksi)</td>
</tr>
<tr>
<td>fps</td>
<td>calculated stress in prestressing reinforcing at the ultimate condition (ksi)</td>
</tr>
<tr>
<td>fy</td>
<td>yield stress of mild reinforcing (ksi)</td>
</tr>
<tr>
<td>Ig</td>
<td>gross moment of inertia of concrete cross-section (in⁴)</td>
</tr>
<tr>
<td>yt</td>
<td>distance from centroid of cross-section to extreme top fiber (in)</td>
</tr>
<tr>
<td>yb</td>
<td>distance from centroid of cross-section to extreme bottom fiber (in)</td>
</tr>
<tr>
<td>b</td>
<td>width of cross-section</td>
</tr>
</tbody>
</table>
Post-Tensioned Concrete Analysis and Design

Design – Notation

rp  - Aps/bdp
St  - I/y_t section modulus w.r.t. top fiber
Sb  - I/y_b section modulus w.r.t. bottom fiber
r  - sqrt(I/Ag)
kt  - -I/AgY_t of S_t/Ag dst. from centroid of the section to the upper limit of the central kern
kb  - I/AgY_b of S_b/Ag dst. from centroid of the section to the lower limit of the central kern
S  - the concrete stress in the top of bottom fiber of the cross-section at a given point
f'_c  - unconfined compressive strength of concrete at 28 days (psi)
f'ci  - unconfined compressive strength of concrete at stressing (psi)
e_o max  - maximum practical tendon eccentricity (in)
h  - percent prestressing stress retained after losses (a decimal)
Mcr  - cracking moment of the cross-section
Ms  - bending moment under superimposed loads at final condition.
Mi  - bending moment under superimposed loads at initial condition.
Mn  - nominal bending moment capacity
Mu  - factored design bending moment.
Example 1: Simple Span Beam RC vs PT

**GIVEN:**
- 50’-0” SIMPLE SPAN BEAM, SIMPLY SUPPORTED
- DEAD LOAD = 1.41klf
- LIVE LOAD = 0.60klf
- $f'_c = 5ksi$
- $f'_d = 3ksi$
- $f_s = 60ksi$

**REQ’D:**
DESIGN THE BEAM FOR FLEXURE —RC & PT

**Example 1: RC Solution**

**Moment w/out Self WT,**

$$M_u = \frac{1.41(1.2) + 0.6(1.6)}{8} (50)^2 = 828.8\text{kft}$$

**Minimum RC Depth by ACI**

318-14 (9.3.1) = $\frac{l}{16} = 37.5\”, \text{USE 38”}$

**Try 18’ Width**

$$M_{sw} = \frac{18(38)}{144} (0.15) (50)^2 = 222.7\text{kft}(1.2) = 267.2\text{kft}$$

**Design Moment,** $M_u = 1096\text{kft}$

**Service Moment,** $M_s = 851\text{kft}$
Example 1: RC Solution

\[ \Phi M_N \geq M_u \]

and

\[ \Phi M_N = \Phi Asf_y(d - \frac{a}{2}) = \Phi 0.85 f'_c ab(d - \frac{a}{2}) \]

substituting

\[ 2d \pm \sqrt{(2d)^2 - \frac{4(31.373)M_u}{f'_c b}} \]

\[ a^2 = \frac{2}{2} \]

because

\[ 0.85 f'_c ab = f_y As \]

then

\[ As = \frac{0.85 f'_c ab}{f_y} \]

The same basic concept will be used to solve for ultimate strength in post-tensioned design...
**Example 1: RC Solution**

**SELECT REINFORCING**

As req’d = 7.53 in²

MAXIMUM # OF BARS IN 1 ROW…. 18”-2(1.5)-2(0.5)=14”

BARS MUST BE SPACED AT $d_b$ or 1”

**5** #11 = (5)1.56 = 7.8 in² **OK**

Clear Spacing Provided = (14”-(5)1.41)/4 = 1.74” **OK**

(Check spacing factor and skin reinforcing.)
**Example 1: RC Solution**

**CHECK DEFLECTION**

\[
\Delta = \frac{5wl^4}{384EI}
\]

where

\[\begin{align*}
wd &= 2.12 \text{ klf} \\
wl &= 0.6 \text{ klf} \\
l &= (50)12 = 600 \text{ in} \\
E &= 4031 \text{ ksi} \\
I_e &= 41295 \text{ in}^4
\end{align*}\]

**Deflection Calculations**

<table>
<thead>
<tr>
<th>As Provided</th>
<th>\text{7.8 sqin.} &lt;-------- INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;a&quot; - As prov'd</td>
<td>6.118 in &lt;-------- INPUT &lt;-------- INPUT</td>
</tr>
<tr>
<td>E concrete =</td>
<td>4030.51 ksi</td>
</tr>
<tr>
<td>E Steel =</td>
<td>29000 ksi &lt;-------- INPUT</td>
</tr>
<tr>
<td>n (Es/Ec) =</td>
<td>7.20</td>
</tr>
<tr>
<td>nAs =</td>
<td>56.12 sqin</td>
</tr>
<tr>
<td>Beta 1 =</td>
<td>0.8 see above</td>
</tr>
<tr>
<td>c =</td>
<td>7.647 code neutral axis, in. (ACI 318 -10.2.7.1 / 22.2.2.4.1)</td>
</tr>
<tr>
<td>2nAs/b =</td>
<td>6.24</td>
</tr>
<tr>
<td>c (computed) =</td>
<td>12.042 computed depth of neutral axis, inches</td>
</tr>
<tr>
<td>MCR =</td>
<td>191.45 Cracking Moment, kft</td>
</tr>
<tr>
<td>Ig =</td>
<td>82308.00 Gross Moment of Inertia, in^4</td>
</tr>
<tr>
<td>&quot;c&quot; comp, Icr =</td>
<td>40822.50 Cracked Moment of Inertia, in^4</td>
</tr>
<tr>
<td>Ie =</td>
<td>41294.85 Equivalent Moment of Inertia, in^4</td>
</tr>
<tr>
<td>&quot;c&quot; from 318, Icr =</td>
<td>45583.19 Cracked Moment of Inertia, in^4</td>
</tr>
<tr>
<td>Ie =</td>
<td>46001.34 Equivalent Moment of Inertia, in^4</td>
</tr>
</tbody>
</table>

**IMMEDIATE DEFLECTIONS** (long term not considered here...)

\[
\begin{align*}
D_{dl} &= 1.79" \quad I/335 \\
D_{ll} &= 0.51" \quad I/1184 \\
D_{total} &= 2.3" \quad I/260
\end{align*}
\]
Example 1: PT Solution

**DESIGN CLASS U BEAM**

**INITIAL TRIAL SECTION DIMENSIONS**

SPAN TO DEPTH RATIO OF L/20 = 30”

BEAM WIDTH = 18”

**SECTION PROPERTIES**

\[ \begin{align*}
I_g &= \frac{bh^3}{12} = 40,500 \text{ in}^4 \\
A_c &= bh = 540 \text{ in}^2 \\
y_t &= y_b = 15” \\
S_t &= I / y_t = 2700 \text{ in}^3 \\
S_b &= I / y_b = 2700 \text{ in}^3
\end{align*} \]

\[ r = \sqrt{\frac{I}{A}} = 8.66 \]

\[ k_t = \frac{-I}{Acy_t} = -5.0” \]

\[ k_b = \frac{I}{Acy_b} = 5.0” \]

**CRITICAL DESIGN MOMENTS**

**DEAD LOAD BENDING MOMENT**

\[ M_{DL+SW} = 1.41klf + \frac{18(30)(0.15)}{144} = \frac{1.973(50)^2}{8} = 616.4 \text{ kft} \]

**LIVE LOAD BENDING MOMENT**

\[ M_{LL} = \frac{0.60(50)^2}{8} = 187.5 \text{ kft} \]

**DESIGN MOMENT,** \[ M_u = 1040 \text{ kft} \]

**SERVICE MOMENT,** \[ M_s = 804 \text{ kft} \]
**Example 1: PT Solution**

**ALLOWABLE STRESSES – ACI 318-14 SECTION 24.5**

**Initial Condition**
Initial concrete compressive stress limit,
\[ \bar{\sigma}_{ci} = 0.60 f_{ci} = 0.6(3000) = 1800 \text{ psi} \]
Initial concrete tensile stress limit,
\[ \bar{\sigma}_{ti} = -3 \sqrt{f_{ci}} = -3 \sqrt{3000} = -164.3 \text{ psi} \]

**Service Condition**
Concrete compressive stress limit,
\[ \bar{\sigma}_{cs} = 0.45 f_{c} = 0.45(5000) = 2250 \text{ psi} \]
Concrete tensile stress limit,
\[ \bar{\sigma}_{ts} = -6 \sqrt{f_{c}} = -6 \sqrt{5000} = -424.3 \text{ psi} \]
Example 1: PT Solution

EXAMINE STRESSES @ POINT OF CRITICAL MOMENT (MIDSPAN)

**Initial Stress**

\[
\sigma_{\text{top}} = \frac{F_i}{Ac} + \frac{F_{e,o} y_t}{I} + \frac{M_{SW} y_t}{I} \quad (\bar{\sigma}_{\text{ii}}) \quad (a)
\]

\[
\sigma_{\text{bottom}} = \frac{F_i}{Ac} + \frac{F_{e,o} y_b}{I} - \frac{M_{SW} y_b}{I} \quad (\bar{\sigma}_{\text{ei}}) \quad (b)
\]

**Final Stress**

\[
\sigma_{\text{top}} = \frac{F_e}{Ac} - \frac{F_{e,o} y_t}{I} + \frac{M_{SW} y_t}{I} + \frac{M_{CDL} y_t}{I} + \frac{M_{LL} y_t}{I} \quad (\bar{\sigma}_{\text{cs}}) \quad (c)
\]

\[
\sigma_{\text{bottom}} = \frac{F_e}{Ac} + \frac{F_{e,o} y_b}{I} + \frac{M_{SW} y_b}{I} + \frac{M_{CDL} y_b}{I} - \frac{M_{LL} y_b}{I} \quad (\bar{\sigma}_{\text{ts}}) \quad (d)
\]

\[
F_e = \text{EFFECTIVE PRESTRESS FORCE AFTER LOSSES}
\]

\[
F_i = \text{INITIAL PRESTRESS FORCE}
\]
Example 1: PT Solution

CENTRAL KERN, k

The region within which an axial compressive force of any magnitude will not produce any tension in the section. (Conceptually similar to keeping the resultant of the soil pressure profile within the center third of a footing with eccentric load.)

\[ \sigma = \frac{F}{A_c} \pm \frac{F_{e_0}y}{I} \geq 0 \quad \text{therefore} \quad \frac{F}{A_c} \geq \frac{F_{e_0}y}{I} \]

LIMIT KERN, k’

The region within which an axial compressive force of a given magnitude can be placed while none of the allowable stresses (tension or compression) are violated.

\[ \sigma_{actual} = \frac{F}{A_c} \pm \frac{F_{e_0}y}{I} \geq \sigma_{allowable} \quad \text{therefore} \quad \frac{F}{A_c} \geq \sigma_{allowable} \pm \frac{F_{e_0}y}{I} \]

- With no externally applied forces, these regions are constant along the length of a member of constant cross-section.

- If external forces are applied, the location of the kern (central and limit) varies with the magnitude of the resulting stresses.
Example 1: PT Solution

CREATE THE LIMITING INEQUALITY EQUATIONS*
Manipulate Equation (a)
\( M_{SW} = M_{\text{min}} = \) Moment due to load at time of stressing

\[
\sigma_{\text{top}} = \frac{F_t}{Ac} + \frac{F_{e,y} y_t}{I} + \frac{M_{SW} y_t}{I} \geq (\sigma_{\text{ult}})
\]
so then,

\[
\frac{F_{e,y} y_t}{I} \geq \sigma_{\text{ult}} - \frac{F_t}{Ac} - \frac{M_{\text{min}} y_t}{I}
\]
isolating

\[
e_o = \frac{\sigma_{\text{ult}} I}{y_t F_t} - I - \frac{M_{\text{min}}}{F_t}
\]
multiply by \((-1)\) and substitute

\[
e_o \leq \frac{\sigma_{\text{ult}} S}{F_t} + k_b + \frac{M_{\text{min}}}{F_t}
\]
and finally,

\[
e_o \leq k_b + (\frac{1}{F_t})(M_{\text{min}} - \sigma_{\text{ult}} S)
\]

Example 1: PT Solution

DETERMINE POST TENSIONING FORCE WITH AID OF INEQUALITY EQUATIONS*

M_{\text{min}} = \text{Moment due to load at time of stressing}
M_{\text{max}} = \text{Moment due to all loads}

\begin{align*}
\text{From}(a) & \quad e_o \leq k_b + \left( \frac{1}{F_i} \right) \left( M_{\text{min}} - \bar{\sigma}_{ts} S_t \right) \quad \text{I} \\
\text{From}(b) & \quad e_o \leq k_i + \left( \frac{1}{F_i} \right) \left( M_{\text{min}} + \bar{\sigma}_{ci} S_b \right) \quad \text{II} \\
\text{From}(c) & \quad e_o \geq k_b + \left[ \frac{1}{F_\varepsilon \text{ (or } \eta Fi)} \right] \left( M_{\text{max}} - \bar{\sigma}_{cs} S_t \right) \quad \text{III} \\
\text{From}(d) & \quad e_o \geq k_i + \left[ \frac{1}{F_\varepsilon \text{ (or } \eta Fi)} \right] \left( M_{\text{max}} + \bar{\sigma}_{ts} S_b \right) \quad \text{IV}
\end{align*}

h = 1 – PT loss due to Shrinkage, Creep, Elastic Shortening, Relaxation, & Friction. (as a decimal)
For this example, \( \eta = 0.875 \) (equivalent to 25ksi)

Example 1: PT Solution

TO PLOT A FEASIBLE REGION

<table>
<thead>
<tr>
<th>Stress Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$e_o \leq 5.0 + \left( \frac{1}{F_i} \right) (616.4(12 - 0.16(2700))$</td>
</tr>
<tr>
<td>II</td>
<td>$e_o \leq 5.0 + \left( \frac{1}{F_i} \right) 7839.6$</td>
</tr>
<tr>
<td>III</td>
<td>$e_o \geq 5.0 + \left( \frac{1}{\eta F_i} \right) (803.9(12) - 2.25(2700))$</td>
</tr>
<tr>
<td>IV</td>
<td>$e_o \geq 5.0 + \left( \frac{1}{\eta F_i} \right) 4082.1$</td>
</tr>
<tr>
<td></td>
<td>$e_o \geq 5.0 + \left( \frac{1}{\eta F_i} \right) (803.9(12) + 0.424(2700))$</td>
</tr>
<tr>
<td></td>
<td>$e_o \geq 5.0 + \left( \frac{1}{F_i} \right) 9716.6$</td>
</tr>
</tbody>
</table>
Example 1: PT Solution

Eccentricity, $e_o$

(1/$F_i$)
Example 1: PT Solution

ANALYTICAL SOLUTION FOR MINIMUM PRESTRESSING FORCE

Intersection of stress condition IV with the point of maximum practical eccentricity.

\[ \text{set } e_{o \max} = \text{condition IV} \]
\[ 13.2'' = 5.0 + \left( \frac{1}{F_i} \right)9716.6 \]
\[ \left( \frac{1}{F_i} \right) = 1.87 \times 10^{-3} \]
\[ F_i = 534.2^{\text{kips}} \]
\[ \text{then} \]
\[ F_e = \eta F_i = 467.4^{\text{kips}} \]
Example 1: PT Solution

Estimation of total tendons required to achieve $F_e = 467$ kips

- **Grade 270 unbonded tendons**, $f_{pu} = 270$ ksi
- **½” dia. Seven wire strands**, $A_{ps} = 0.153$ in$^2$ / strand
- **Low Relaxation**, $f_{py} = 0.9f_{pu} = 243$ ksi (ACI 318-14, Table R20.3.2.3.1)
- **Max. force at stressing**, $0.94f_{py} \leq 0.80f_{pu} = 228.4$ ksi $\leq 216$ ksi (Table 20.3.2.5.1)
- **Initial force (max)**, $f_{pi} = 0.7f_{pu} = 189$ ksi (Table 20.3.2.5.1)
- $\eta_{assumed} = 0.875$ therefore $f_{pe} = \eta f_{pi} = 189$ksi
- $A_{ps} = \frac{467.4^k}{189}$ ksi $= \frac{2.473}$ in$^2$ $= 16.16$ strands

**Use 17 strands**; $A_{ps} = 0.153$ in$^2$ (17) $= 2.6$ in$^2$
Example 1: PT Solution

Complete Tendon Profile

AT MIDSPAN;  
\[ e = 13.2'' \]  \( \text{Fi} = 534.2 \text{k} \quad \text{Fe} = 467.4 \text{k} \)

\[
\sigma_{gi} = \frac{Fi}{Ac} = \frac{534.2}{540} = 0.989 \text{ ksi}
\]

\[
\sigma_{g} = \frac{F}{Ac} = \frac{467.4}{540} = 0.866 \text{ ksi}
\]

\[
k_{i}^{1} = \text{LARGER OF}
\]

\[
k_{b} (1 - \frac{\bar{\sigma}_{cs}}{\sigma_{g}}) = 5.0 \left(1 - \frac{2.25}{0.866}\right) = -8.0''
\]

\[
k_{t} (1 - \frac{\bar{\sigma}_{ts}}{\sigma_{g}}) = 5.0 \left(1 - \frac{0.424}{0.866}\right) = -7.45''
\]

\[
k_{i}^{1} = \text{SMALLER OF}
\]

\[
k_{b} (1 - \frac{\bar{\sigma}_{ti}}{\sigma_{gi}}) = 5.0 \left(1 - \frac{0.164}{0.989}\right) = 5.83''
\]

\[
k_{t} (1 - \frac{\bar{\sigma}_{ci}}{\sigma_{gi}}) = 5.0 \left(1 - \frac{1.80}{0.989}\right) = 4.10''
\]

Calculation of Limit Kern
## Example 1: PT Solution

<table>
<thead>
<tr>
<th>DISTANCE ALONG SPAN</th>
<th>0’</th>
<th>5’</th>
<th>10’</th>
<th>15’</th>
<th>20’</th>
<th>25’</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\text{MIN}}$ (\text{ink})</td>
<td>0</td>
<td>2663.6</td>
<td>4735.2</td>
<td>6215.0</td>
<td>7102.8</td>
<td>7396.8</td>
</tr>
<tr>
<td>$M_{\text{MAX}}$ (\text{ink})</td>
<td>0</td>
<td>3473.6</td>
<td>6175.2</td>
<td>8105.0</td>
<td>9262.8</td>
<td>9646.8</td>
</tr>
<tr>
<td>$M_{\text{MIN}} / Fi$ (\text{in})</td>
<td>0</td>
<td>4.986</td>
<td>8.864</td>
<td>11.634</td>
<td>13.296</td>
<td>13.846</td>
</tr>
<tr>
<td>$M_{\text{MAX}} / Fe$ (\text{in})</td>
<td>0</td>
<td>7.432</td>
<td>13.212</td>
<td>17.34</td>
<td>19.82</td>
<td>20.639</td>
</tr>
<tr>
<td>$e_{ai} = k_i + \frac{M_{\text{MAX}}}{F}$</td>
<td>-7.45”</td>
<td>-0.02”</td>
<td>5.76”</td>
<td>9.89”</td>
<td>12.37”</td>
<td>13.19”</td>
</tr>
<tr>
<td>$e_{ai} = k_i b + \frac{M_{\text{MIN}}}{Fi}$</td>
<td>4.10”</td>
<td>9.09”</td>
<td>12.96”</td>
<td>15.73”</td>
<td>17.40”</td>
<td>17.95”</td>
</tr>
<tr>
<td>$e_{ai}$ (\text{in})</td>
<td>0”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$M_{\text{MIN}} = M_{\text{DL+SW}}$  \hspace{1cm}  $M_{\text{MAX}} = M_{\text{DL+SW}+M_{\text{LL}}}$
Example 1: PT Solution

Region of allowable tendon profile at $F_e = 467.4$ k

Beam C.G.

Midspan solution point

Eccentricity, $e_o$

Distance along span (ft)
Example 1: PT Solution

ULTIMATE STRENGTH

CALCULATE $\Phi M_n$ FOR PRESTRESSED MEMBERS IN SAME MANNER AS REINFORCED CONCRETE MEMBER, SUBSTITUTE $f_{ps}$ FOR $f_y$

AS ALTERNATE TO DETERMINING $f_{ps}$ BY STRAIN COMPATIBILITY, APPROXIMATE VALUES FROM 20.3.2.3 and 20.3.2.4 MAY BE USED.

• BONDED TENDONS

$$f_{ps} = f_{pu} \left\{ 1 - \frac{\gamma_p}{\beta_1} \left[ \rho_p \frac{f_{pu}}{f'c} + \frac{d}{dp} \frac{f_y}{f'c} (\rho - \rho') \right] \right\}$$

• UNBONDED TENDONS

If span/depth ≤ 35

$$f_{ps} = f_{pe} + 10000 + \frac{f'c}{100\rho_p} < f_{py} < (f_{pe} + 60000)$$

If span/depth > 35

$$f_{ps} = f_{pe} + 10000 + \frac{f'c}{300\rho_p} < f_{py} < (f_{pe} + 30000)$$
Example 1: PT Solution

**ULTIMATE STRENGTH**

\[ A_{ps} = 2.473 \text{ in}^2 \]

\[ \rho = \frac{A_{ps}}{bd} = \frac{2.473}{18(28.2)} = 0.00487 \]

so:  
\( \text{because } 50''/(30''/12) = 20 \)

\[ f_{ps} = 189000 \text{ psi} + 10000 + \frac{5000}{100(0.00487)} = 209.3 \text{ ksi} \]

then:

\[ a = \frac{A_{ps}f_{ps}}{0.85 f'_c b} = \frac{2.473(209.3)}{0.85(5)(18)} = 6.766'' \]

\[ \Phi M_n = 0.9(0.85 f'_c ab(d - \frac{a}{2})) = 0.9((0.85)(5)(18)(6.77)(28.2 - \frac{6.77}{2})) \]

\[ \Phi M_n = 11566.6 \text{ in}k = 963.9 \text{ fik} < 1040 \text{ fik} \quad \text{... we are not done yet,} \]

ADD SUPPLEMENTARY MILD REINFORCING.
Example 1: PT Solution

ULTIMATE STRENGTH – SUPPLEMENTARY MILD REINFORCING

INTERNAL COMPRESSION FORCE = INTERNAL TENSION FORCE

\[ 0.85 f' c \ ab = Apsfps + Asfy \] ①

\[ \Phi Mn = \Phi (Apsfps (dp - \frac{a}{2}) + Asfy (d - \frac{a}{2})) \] ②

rearranging ①, \[ a = \frac{Apsfps + Asfy}{0.85 f' c \ b} \] ③

Substituting ③ into ②, and 2 Pages of Algebra Later...

\[ \Phi Mn = \Phi \{Apsfpsdp[1 - 0.6(\frac{\rho_{ps}fps}{f' c} + \frac{d \ \rho_{fy}}{f' c})] + Asfyd[1 - 0.6(\frac{dp \ \rho_{ps}fps}{d \ f' c} + \frac{\rho_{fy}}{f' c})] \} \]

Substituting \[ \frac{As}{bd} = \rho \] to Solve for \[ As_{\text{req'd}} \]  ... Later,

\[ As^2 (\frac{-0.6fy^2}{bf' c}) + As (fyd - \frac{1.2 Apsfpsfy}{bf' c}) + (Apsfpsdp - \frac{0.6 Aps^2 fps^2}{bf' c}) - \frac{Mu}{\Phi} = 0 \]

mult. by bf' c and...

\[ \text{A} \quad \text{B} \quad \text{C} \]
# Example 1: PT Solution

**ULTIMATE STRENGTH – SUPPLEMENTARY MILD REINFORCING**

<table>
<thead>
<tr>
<th>Calculation of Mild Steel Reinforcement Required for Ultimate Strength of Post Tensioned Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>(all units must be in pounds and inches)</td>
</tr>
<tr>
<td>$F_y = 60000 \text{ psi}$</td>
</tr>
<tr>
<td>$A_{ps} = 2.47301587 \text{ in}^2$</td>
</tr>
<tr>
<td>$f_{ps} = 209262.773 \text{ psi}$</td>
</tr>
<tr>
<td>rebar depth = 27.3 in</td>
</tr>
<tr>
<td>$b \text{ (flange)} = 18 \text{ in}$</td>
</tr>
<tr>
<td>$f_{c} = 5000 \text{ psi}$</td>
</tr>
<tr>
<td>tendon depth = 28.2 in</td>
</tr>
<tr>
<td>$Mu = 12480000 \text{ lbin}$</td>
</tr>
<tr>
<td>$\phi = 0.9$</td>
</tr>
<tr>
<td>$f_{se} = 189000 \text{ psi}$</td>
</tr>
<tr>
<td>$\text{Astrand} = 0.153 \text{ in}^2$</td>
</tr>
<tr>
<td>force/strnd = 28917.00 lbs</td>
</tr>
<tr>
<td>slab thk = 0 in</td>
</tr>
<tr>
<td>$b \text{ (web)} = 18 \text{ in}$</td>
</tr>
</tbody>
</table>

This spreadsheet is for use when the compression block is less than the slab/flange depth.

Use (2) #6, $A_{prov'd} = 0.88 \text{ in}^2$
Example 1: PT Solution

ULTIMATE STRENGTH – SUPPLEMENTARY MILD REINFORCING

CHECK MINIMUM BONDED REINFORCEMENT  
(ACI 318-14, 9.6.2.3)

As = 0.004A  
A = \frac{1}{2} Ac  
(for symmetric section)

As = 1.08 > 0.88, use (3) #6.

This requirement only applies to members with unbonded tendons...

CHECK CRACKING MOMENT  
(ACI 318-14, 9.6.2.1)

\Phi M_n \geq 1.2 Mcr

fr = 7.5 \sqrt{f'_c}

Mcr = \frac{frIg}{yt} = 1,431,891.2 \text{in}\cdot\text{lb} = 1431.9 \text{in}k = 119.3 \text{fik}

1040 >> 143.2

This requirement does not apply for members with unbonded tendons...
Example 1: PT Solution

ULTIMATE STRENGTH – BONDED TENDONS

\[ fp_s = f_{pu} \left\{ 1 - \frac{\gamma_p}{\beta_1} \left[ \rho_p \frac{f_{pu}}{f'_c} + \frac{d}{dp} \frac{f_y}{f'_c} (\rho - \rho') \right] \right\} \]

\[ \rho_p = \frac{A_{ps}}{bd_p} \]

\[ \gamma_p = 0.28 \quad \text{0.28 FOR LOW RELAXATION STRANDS} \]
\[ = 0.40 \quad \text{0.40 FOR STRESS RELIEVED STRANDS} \]

\[ fp_s = 270 \left\{ 1 - \frac{0.28}{0.80} \left[ \frac{0.00487(270)}{5} + 0 \right] \right\} = 245.15 \text{ ksi} \]

\[ a = \frac{2.47(245.15)}{0.85(18)(5)} = 7.92'' \]

\[ \Phi M_n = 0.9 \left( 0.85(5)(18)(7.92)(28.2 - \frac{7.92}{2}) \right) = 13217.9 \text{ ink} \]

\[ \Phi M_n = 1101.5 \text{ fitk} > 1040 \text{ fitk} \]
Example 1: PT Solution

ELASTIC DEFLECTIONS

**RC SOLUTION**

\[ \Delta = \frac{5wl^4}{384EI_e} \]

where

- \( w_d = 2.12 \) klf
- \( w_l = 0.6 \) klf

\( l = (50)12 = 600 \) in

\( E = 4031 \) ksi

\( I_e = 41295 \) in\(^4\)

**PT SOLUTION**

\[ \Delta = \frac{5(w_{d+1})l^4}{384EI_g} - \frac{5(w_{eq})l^4}{384EI_g} \]

where

- \( w_d = 1.97 \) klf
- \( w_l = 0.6 \) klf

\[ w_{eq} = \frac{8Pe^g}{l^2} = \frac{8(467.4)(13.2)}{600^2} = 0.137 \text{ k} / \text{in} = 1.645 \text{ k} / \text{ft} \]

\( l = (50)12 = 600 \) in

\( E = 4031 \) ksi

\( I_e = I_g = 40500 \) in\(^4\)

**SHORT TERM ELASTIC DEFLECTIONS** (long term not considered here...)

**RC SOLUTION**

- \( D_{dl} = 1.79'' \) l/335
- \( D_{ll} = 0.51'' \) l/1184
- \( D_{total} = 2.3'' \) l/260

**PT SOLUTION**

- \( D_{dl} = 0.28'' \) l/2143
- \( D_{ll} = 0.52'' \) l/1154
- \( D_{total} = 0.8'' \) l/750

For further reading reference ACI 435R.
Post-Tensioned Concrete Analysis and Design

LOSS OF PRESTRESS

ACI 314-15 SECTION 20.3.2.6.1

Prestress losses shall be considered in the calculation of the effective tensile stress in the prestressed reinforcement, fse, and shall include (a) through (f):

a) PRESTRESSED REINFORCEMENT SEATING AT TRANSFER (Seating Loss)
b) ELASTIC SHORTENING OF CONCRETE
c) CREEP OF CONCRETE
d) SHRINKAGE OF CONCRETE
e) RELAXATION OF PRESTRESS REINFORCEMENT
f) FRICTION LOSS DUE TO INTENDED OR UNINTENDED CURVATURE IN POST-TENSIONING TENDONS.
Post-Tensioned Concrete Analysis and Design

LOSS OF PRESTRESS

a) Prestress Reinforcement Seating at Transfer (Seating Loss)

Caused by Mechanical Interactions of Stressing Jack and Anchor.
Can be Reduced by:

- OVERJACKING
- WEDGE SETTERS
Post-Tensioned Concrete Analysis and Design

LOSS OF PRESTRESS

b) Elastic Shortening

PCI HANDBOOK / ACI 435R

\[ ES = K_{es} f_{c_{ir}} \frac{E_{ps}}{E_{ci}} \]

\[ f_{c_{ir}} = K_{c_{ir}} \left( \frac{F_i}{A_g} + \frac{F_i e^2}{I_g} \right) - \frac{M_{ge}}{I_g} \]

Kes = 1.6 for Post-Tensioning

Kes = 2.0 for Prestressed

\( f_{c_{ir}} = 0.548 \text{ ksi} \)

\( E_{ci} = 33 \omega^{3/2} \sqrt{f_{c_{ir}}} = 33(45)^{3/2} \sqrt{3000} = 3155.9 \text{ ksi} \)

and

\( E_{ps} = 28.5 \times 10^6 \text{ psi} \)

so

\[ ES = 0.5(28.5 \times 10^6)(548)/3155900 \]

\[ ES = 2474.4 \text{ psi} \]
c) Creep of Concrete

PCI HANDBOOK –\( CR = K_{cr}(Eps / Ec)(f_{cir} - f_{cds}) \)

- \( K_{cr} = 2.0 \) for Normal Weight Concrete
- \( = 1.6 \) for Sand Light-Weight Concrete
- \( Ec = 28 \) Day Modulus of Elasticity of Concrete
- \( f_{cds} = \) Stress in Concrete at Center of Gravity of Tendons Due to All Superimposed Permanent Dead Loads that are Applied to the Member after it has been Prestressed.

\[ f_{cir} - f_{cds} = 0.548 \text{ ksi} - 0 \quad \leftarrow \text{superimposed dead load initially applied.} \]

\[ Ec = 33w_c^{1.5} \sqrt{f'c} = 4074.3 \text{ ksi} \]

\[ so \]

\[ CR = 7667 \text{ psi} \]
Post-Tensioned Concrete Analysis and Design

LOSS OF PRESTRESS

d) Shrinkage of Concrete

PCI HANDBOOK / ACI 435R

\[ SH = (8.2 \times 10^{-6}) K_{SH} E_{ps} (1 - 0.06V / S)(100 - R.H.) \]

\[ K_{SH} = \text{See Table Below} \]

\[ V / S = \text{Volume to Surface Ratio} \]

\[ R.H. = \text{Average Ambient Relative Humidity, use 70%} \]

<table>
<thead>
<tr>
<th>Time from end of moist curing to application of prestress, days.</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{SH} )</td>
<td>0.92</td>
<td>0.85</td>
<td>0.8</td>
<td>0.7</td>
<td>0.73</td>
<td>0.65</td>
<td>0.58</td>
<td>0.45</td>
</tr>
</tbody>
</table>

\[ V / S = \frac{18(30)}{2(18 + 30)} = 5.625 \]

\[ SH = (8.25 \times 10^{-6})(0.80)(28.5 \times 10^6)(1 - 0.06(5.625))(100 - 70) \]

\[ SH = 3715.8 \text{ psi} \]
Post-Tensioned Concrete Analysis and Design

LOSS OF PRESTRESS

e) Relaxation of Tendons

PCI HANDBOOK –

\[
RE = \left[ K_{RE} - J(SH + CR + ES) \right] C
\]

<table>
<thead>
<tr>
<th>270 KSI Strand</th>
<th>KRE</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRESS RELIEVED</td>
<td>20000</td>
<td>0.15</td>
</tr>
<tr>
<td>LOW RELAXATION</td>
<td>5000</td>
<td>0.040</td>
</tr>
</tbody>
</table>

\[
RE = \left[ 5000 - 0.04(3716 + 7667 + 2474) \right] 1.28
\]

\[
RE = 5690 \text{ psi}
\]
f) Friction Loss

\[ P_{px} = P_{pj} e^{-(K \ell_{px} + \mu p \alpha_{px})} \]

Where \((K \ell_{px} + \mu p \alpha_{px}) \leq 3.0\), Computation by \( P_{px} = P_{pj} (1 + K \ell_{px} + \mu p \alpha_{px})^{-1} \) is acceptable.

FOR 7- WIRE STRANDS

| UNBONDED GREASED: 0.0003-0.0020 | 0.05-0.15 |
| BONDED: 0.0005-0.0020 | 0.15-0.25 |

\( K = \) Wobble Coefficient
\( \mu = \) Curvature Coefficient.

\( \alpha_{px} = \) Tendon Angle, Radians
\( \ell_{px} = \) Length, Anchor to pt.

POST-TENSIONING SUPPLIER PROVIDES FRICTION LOSS CALCULATIONS BASED ON THE TENDON PROFILE SPECIFIED AND MATERIAL PROPERTIES OF THE PRODUCT HE IS PROVIDING.
# Post-Tensioned Concrete Analysis and Design

## LOSS OF PRESTRESS

### Typical Calculation Submittal

<table>
<thead>
<tr>
<th>STRAND PROPERTIES</th>
<th>Ultimate Stress, fpu: 270 ksi</th>
<th>Cross-Sectional Area: 0.153 sq. in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus:</td>
<td>28500 ksi</td>
<td>Strand Type: Grade 270 Low Reaxion</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OTHER INFO</th>
<th>% Jacking</th>
<th>80%</th>
<th>Slab Thickness: 6.0 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seating Loss:</td>
<td>0.25% in.</td>
<td>Beam Height: 30.0 in</td>
<td></td>
</tr>
<tr>
<td>Number of Strands:</td>
<td>7 okl</td>
<td>Beam Width: 14.0 in</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LONG-TERM LOSSES</th>
<th>Average Initial compression per strand at stressing: 28.80 kips (after seating)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P/A:</td>
<td>203 psi</td>
</tr>
<tr>
<td>Concrete Strength, Stressing:</td>
<td>3500 psi</td>
</tr>
<tr>
<td>Concrete Strength (28-day):</td>
<td>5000 psi</td>
</tr>
<tr>
<td>Volume to Surface Ratio:</td>
<td>3.72 in.</td>
</tr>
<tr>
<td>Moist Cure to Stressing:</td>
<td>3 days</td>
</tr>
<tr>
<td>Force in This Strip:</td>
<td>kips</td>
</tr>
<tr>
<td>Relative Humidity:</td>
<td>80%</td>
</tr>
<tr>
<td>Force in Each Strand:</td>
<td>0.5 for post-tensioned members</td>
</tr>
<tr>
<td>Force in Each Strand:</td>
<td>1.6 for post-tensioned members</td>
</tr>
<tr>
<td>Force in Each Strand:</td>
<td>0.65 for 3 days from moist cure to stress</td>
</tr>
<tr>
<td>Force in Each Strand:</td>
<td>5 ksi for 270 grade low-lax strand</td>
</tr>
<tr>
<td>Force in Each Strand:</td>
<td>0.04 for 270 grade low-lax strand</td>
</tr>
<tr>
<td>Force in Each Strand:</td>
<td>0.7 for low-lax strand at 98% Fu</td>
</tr>
</tbody>
</table>

Formulas used are based on Estimating Prestress Losses, Concrete International, June 1979.

- **Eo** = 57000 (°C)^0.5
- **ES** = Kes (P/A) (E/Eci)
- **Creep** = Kc (P/A) (E/Eci)
- **Shrinkage** = S = 8.2 - 0.65 [S - 0.06 (V/S)] (100 - RH)
- **Relaxation** = RE = (Kr - Jre (ES + CR + SH)) Cre

### SUMMARY

- **First Elongation:** 9.10 in./ft, 0.0799
- **Second Elongation:** 0.33 in.
- **Pmax:** 30.4 kips, 0.74 fpu
- **Pave:** 27.6 kips, 0.67 fpu
- **Total force in beam:** 193 kips

1. **Pmax**, **Pmin**, and elongations include seating loss but not long-term losses.
2. **Pave** includes long-term losses.
3. The values for the first and second pulls include the seating loss.
Total Losses for Example 1

Total Losses = ES+CR+SH+RE = 2,474+7,667+3,716+5,690 = 19,547 psi

\[ f_{pi} = 0.80 f_{pu} = 216 \text{ ksi} \]

\[ f_{pe} = f_{pi} - TL = 196.4 \text{ ksi} \text{ but this number does not include friction loss...} \]

From Example 1,
(we did not round, and assumed \( \eta = 0.875 \))

\[ F_i = 534.2 \text{ kips} \]
\[ F_e = n F_i = 0.875(534.2) = 467.4 \text{ kips} \]
\[ f_{pi} = 216 \text{ ksi} \]
\[ f_{pe} = 189 \text{ ksi or about 28.9 kips/ strand} \]

IF - seating loss is 0.25” = 28.5X10^6(0.25) = 7,125 psi

IF - friction loss is in range of other loss amounts or \( \approx 2,400 \text{ psi} \) then \( TL \approx 29,072 \text{ psi} \) which yields
\[ f_{pe} \approx 187 \text{ ksi (28.6 kips / strand)} \text{ or } \eta = 0.866 \]

WHY IS PT STRAND LOSS CALCULATION AND VERIFICATION IMPORTANT?
Post-Tensioned Concrete Analysis and Design

LOSS OF PRESTRESS

WHY IS PT STRAND LOSS CALCULATION AND VERIFICATION IMPORTANT?

• Initial Camber

• Final Deflection

• ULTIMATE STRENGTH
  • Provision of Adequate Post-Tensioning Reinforcement, Aps.
  • Proportioning Mild Reinforcing, As.
Example 2: Continuous Beam

**GIVEN:**
- PT PARKING GARAGE, 3 BAYS, 20'-0” SLAB SPAN
- 54'-0” BEAM SPAN
- 24”x32” COLUMNS (LONG DIM. IN DIRECTION OF FRAME)
- LIVE LOAD = 40 psf
- COLLATERAL DEAD LOAD = 5 psf

\[ f'c = 5000 \text{ psi} \]
\[ f'ci = 3000 \text{ psi} \]
\[ f_y = 60 \text{ ksi} \]
\[ f_{uy} = 270 \text{ ksi, } 7/8 \phi 7 - \text{wire strands} \]

**REQ’D:** DESIGN THE BEAM

ASSUME 5¼ SLAB (then to fit the beam bars into the column...)
ASSUME BEAM DIMENSIONS = 24-1.5-1.5-0.5-0.5-2(1.0)= 18” WIDTH
54/2 = 27” DEPTH

**DEAD LOAD** = 5psf+Self Wt. = \( 100 p \ell f + \frac{5\frac{1}{4}(150)(20)}{12} + \frac{18(27 - 5\frac{3}{4})(150)}{144} = 1.82k \ell f \)

\[ = 1.72k \ell f \text{ (Self Weight ONLY)} \]

**LIVE LOAD** = 40psf => \( 40(20) = 800 p \ell f = 0.8 k \ell f \)
Example 2: Continuous Beam

T-BEAM PROPERTIES

\[ 8t_f = 42'' \quad \text{TOTAL} = 42(2) + 18 = 102 \]

\[ \frac{h}{4} \ell = 162'' \]

\[ \frac{h}{2} \ell_T = 120'' \]

\[ y = \frac{102(5\frac{3}{4})(27 - 5\frac{5}{2}) + (27 - 5\frac{3}{4})(18)(\frac{27-5\frac{3}{4}}{2})}{102(5\frac{3}{4}) + (27 - 5\frac{3}{4})(18)} = 18.67'' \]

\[ Ig = \frac{18(27 - 5\frac{3}{4})}{12} + 18(27 - 5\frac{3}{4})(18.67 - 10.88)^2 + \frac{102(5\frac{3}{4})^3}{12} + 102(5\frac{3}{4})(24.38 - 18.67)^2 = 57,881 \text{in}^4 \]

SECTION PROPERTIES

\[ \text{Area} = 102(5\frac{3}{4}) + (27 - 5\frac{3}{4})(18) = 927 \text{in}^2 \]

\[ Ig = 57881 \text{in}^4 \]

\begin{align*}
\text{DISTANCE FROM CENTROID TO BOTTOM FIBER} & \quad y_b = 18.67'' \\
\text{DISTANCE FROM CENTROID TO TOP FIBER} & \quad y_t = 8.33'' \\
\text{SECTION MODULUS} & \quad S_t = \frac{y_t}{y_b} = 3100.2 \text{in}^3 \\
\text{SECTION MODULUS} & \quad S_b = \frac{y_b}{y_t} = 6948.5 \text{in}^3 \\
\text{RADIUS OF GYRATION} & \quad r = \sqrt{\frac{S_t}{A}} = 7.90 \\
\text{DISTANCE FROM CENTROID TO UPPER LIMIT OF CENTRAL KERN} & \quad k_t = \frac{r}{A_y} = -3.34'' \\
\text{DISTANCE FROM CENTROID TO LOWER LIMIT OF CENTRAL KERN} & \quad k_b = \frac{r}{A_y} = 7.5''
\end{align*}
Example 2: Continuous Beam

DETERMINE CRITICAL MOMENTS

INITIAL FLEXURAL MOMENTS (SELF WEIGHT ONLY)

SERVICE FLEXURAL MOMENTS (SELF WEIGHT + SUPERIMPOSED DEAD LOAD + LIVE LOAD)
Example 2: Continuous Beam

SOLVE FOR PT FORCE & PROFILE BY LOAD BALANCING

1. SELECT THE BALANCED LOAD, Wb.
   Determine % of Live Load to Balance,

   \[ M = 0.1wl^2 \]  
   (Maximum negative moment for three span continuous beam.)

   and

   \[ \sigma = \frac{My}{I} \]  
   (Keep the bending stress below allowable tensile limit.)

   so

   \[ \frac{0.10wl^2(8.33)}{57881} = 0.424 \]

   \[ w = 0.07\text{kip/inch} = 0.842 \text{kip/ft} \]

   Full Live Load = 0.80

   Therefore only balance the dead load, balancing of live load is not required to meet tensile stress limits. \( \underline{Wb=1.82 \text{ klf}} \)
Example 2: Continuous Beam

SOLVE FOR PT FORCE & PROFILE BY LOAD BALANCING

2. SELECT STEEL PROFILE (PARABOLIC) WITH MAXIMUM PRACTICAL ECCENTRICITIES. (27” DEEP BEAM, 5¼” SLAB)
Example 2: Continuous Beam

SOLVE FOR PT FORCE & PROFILE BY LOAD BALANCING

3. DETERMINE THE PRESTRESSING FORCE REQUIRED IN EACH SPAN TO BALANCE Wb OF THAT SPAN.

a) Middle Span

\[ 1.82 \frac{klf}{l^2} = \frac{8Pe}{l^2} = \frac{8P(23.38''/12)}{54^2} \]

\[ P = 340.5 \text{kips} \]

b) End Spans – left side of span force must equal right side.

\[ \frac{8Pe'_1}{[2(xl)]^2} = \frac{8Pe'_2}{[2(1-x)l]^2} \]

if

\[ e'_1 = 16.86'' \quad \text{and} \quad e'_2 = 23.88'' \]

then

\[ P = 398.3 \text{kips} \]

\[ 1.82 \frac{klf}{l^2} = \frac{8Pe'_1}{[2(xl)]^2} = \frac{8P(16.86/12)}{(2(0.4592(54))^2} \]

\[ x = 0.4592 \]

\[ 16.86'' = \frac{23.88''}{[2(54(12))x]^2} = \frac{23.88''}{[2(54(12)(1-x))]^2} \]

solve quadratic,

\[ x = 0.4592 \]

c) Adjust Profile in Center Span. \ e = 19.98'', \text{ USE 20''} \]
Example 2: Continuous Beam

SOLVE FOR PT FORCE & PROFILE BY LOAD BALANCING

4. COMPUTE MOMENTS CAUSED BY REMAINING LOAD OR UNBALANCED LOAD ON THE FRAME. (Determined Previously)

5. MODIFY THEORETICAL TENDON PROFILE TO REVERSE PARABOLIC PROFILE. (Calculate Equivalent Loads and Solve Frame with PT Loads Only)

\[ w_1 = \frac{8Pe}{l^2} = 1.82 \text{ klf} \]
\[ w_2 = \frac{w_4}{b} (0.5 - b) = 9.08 \text{ klf} \]
\[ w_3 = \frac{w_5}{b} (0.5 - b) = 9.08 \text{ klf} \]
\[ w_4 = \frac{4Pe}{(0.5 - b)l^2} = 2.27 \text{ klf} \]
\[ w_5 = \frac{4Pe}{(0.5 - b)l^2} = 2.27 \text{ klf} \]
Example 2: Continuous Beam

Moments from Post-Tensioning Force.

(with no restraint from supports)

(these moments include secondary effects of restraint of supports)
### Example 2: Continuous Beam

#### Moment Summary

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dead Load Moments</strong></td>
<td>-381</td>
<td>219</td>
<td>-435</td>
<td>-419</td>
<td>208</td>
<td>-419</td>
<td>-435</td>
<td>219</td>
</tr>
<tr>
<td><strong>Final Moments</strong></td>
<td>-580</td>
<td>333</td>
<td>-662</td>
<td>-638</td>
<td>317</td>
<td>-638</td>
<td>-662</td>
<td>333</td>
</tr>
<tr>
<td><strong>Equivalent Loading</strong> (P&lt;sub&gt;eff&lt;/sub&gt;)</td>
<td>430</td>
<td>-260</td>
<td>401</td>
<td>397</td>
<td>-265</td>
<td>401</td>
<td>397</td>
<td>-260</td>
</tr>
<tr>
<td><strong>Initial Moments</strong>  (w/ P&lt;sub&gt;i&lt;/sub&gt;=P&lt;sub&gt;eff&lt;/sub&gt;/0.80)</td>
<td>156</td>
<td>-106</td>
<td>66</td>
<td>77</td>
<td>-123</td>
<td>77</td>
<td>66</td>
<td>-106</td>
</tr>
<tr>
<td><strong>Service Moments</strong></td>
<td>-150</td>
<td>73</td>
<td>-261</td>
<td>-241</td>
<td>52</td>
<td>-241</td>
<td>-261</td>
<td>73</td>
</tr>
</tbody>
</table>
Example 2: Continuous Beam

Check Critical Stresses at Service – Class U Design

\[ \sigma_{T/B} = \frac{P}{Ac} \pm \frac{Pe}{S} \pm \frac{(M_s + M_2)}{S} \]

\[ \bar{\sigma}_{ti} = -3\sqrt{f'_c} = -164.3 \text{ psi} \]

Initial Top and Bottom Stress Limits

\[ \bar{\sigma}_{ci} = 0.6 f'_{ci} = 1,800 \text{ psi} \]

Service / Final Top and Bottom Stress Limits

\[ \bar{\sigma}_{ts} = -6\sqrt{f''_c} = -424.3 \text{ psi} \]
Example 2: Continuous Beam

Stress Summary

\[
\frac{P}{Ac} = \frac{398.3}{927} = 0.430 \text{ ksi} = 430 \text{ psi}
\]

\[
y_t = 8.33"
\]

\[
S_t = 3,100.2 \text{ in}^3
\]

\[
y_b = 18.67"
\]

\[
S_b = 6,948.5 \text{ in}^3
\]

<table>
<thead>
<tr>
<th>INITIAL STRESSES (w/ ( P_i = P_{eff}/0.80 )), psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1141</td>
</tr>
<tr>
<td>268</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SERVICE STRESSES, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>-151</td>
</tr>
<tr>
<td>689</td>
</tr>
</tbody>
</table>

Service Interior Negative Moment at Face of Column = 189 kft

\[
\sigma = 430 - \frac{(12(1000(189)))}{3100.2} = -301 \text{ psi} \quad \text{OK}
\]
Denouement – Secondary Moments and Ultimate Strength

ACI 318-08, Section 18.10.3

“Moments used to compute required strength shall be the sum of the moments due to reactions induced by prestressing (with a load factor of 1.0) and the moments due to factored loads.”

\[ \text{Mu} = 1.2DL + 1.6LL + 1.0 M_2 \]
Example 2: Continuous Beam

Secondary Moments and Ultimate Strength

\[ M_{\text{total}} - \text{Apply Equivalent loads to Frame.} \]

\[ M_{\text{primary}} - \text{Apply Equivalent Loads to Unrestrained Beam.} \]

\[ M_{\text{secondary}} - \text{Subtract } M_{\text{primary}} \text{ from } M_{\text{total}} \]
Example 2: Continuous Beam

Secondary Moments and Ultimate Strength – Alternate Look

Apply Equivalent loads to frame to obtain Hyperstatic Reactions.

Apply Hyperstatic Reactions to Unrestrained Beam to Obtain...

\[ M_{\text{secondary}} \]

(This is what PTI calls the “Direct Method”)

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Post-Tensioned Concrete Analysis and Design

- Common Materials & Properties / Basics
  - Common Prestressing Material and Strengths
  - Bonded and Unbonded Systems / Construction Sequence
- General Post-Tensioned Concrete Framing Parameters
  - Span-to-Depth Ratios / Deflection
  - Range of Prestress Force and the Effective Flange
  - Classes of Prestressed Concrete Members
- Design Concept
  - Feasible Region
  - Load Balancing
  - Equivalent Load
- Steps in Post-Tensioned Concrete Design
- Example 1 – Simple Beam Design (Graphical Solution / Feasible Region)
  - Conventional Reinforced Concrete Solution
  - Post-Tensioned Solution
- Post-Tensioning Losses
- Example 2 – Continuous Beam Design (Load Balancing and Equivalent Loading)
Learning Objectives

- Identify how to determine long term prestress force loss and its effect on service level and ultimate strength design.
- Recognize the effects of secondary post-tensioning forces on frame construction.
- Determine member size and prestress force using the feasible region method and understand load balancing and equivalent loading.
Post-Tensioned Concrete Analysis & Design

Otto J. Schwarz, P.E., S.E.

Ryan Biggs | Clark Davis Engineering and Surveying, D.P.C.

SE University, October 2017
CHALLENGE QUESTION:

Which Design Concept related to Post-Tensioned Concrete is the answer to this session’s Challenge Question?

A. Feasible Region
B. Load Balancing
C. Equivalent Load
D. Secondary Moment

Please circle the answer that is announced so that you can use the information to complete your quiz for the Attendance Verified PDH.